TD 03

Exercise 1

Let P^* be the set of prime numbers strictly greater than 2.
We consider the relation $\mathcal{R}$ between two elements of P^* defined as :

$$p\mathcal{R}q \Leftrightarrow \frac{p+q}{2} \in P^*.$$

Is the relation $\mathcal{R}$ reflexive, symmetric, and transitive ?

Exercise 2

Let $\mathcal{R}$ be a relation defined on $\mathbb{Z} \times \mathbb{N}^*$ as :

$$(a,b)\mathcal{R}(a',b') \Leftrightarrow ab' = a'b.$$

1. Show that $\mathcal{R}$ is an equivalence relation.
2. Let $(p,q) \in \mathbb{Z} \times \mathbb{N}^*$, with $\gcd(p,q) = 1$. Describe the equivalence class of (p,q).

Exercise 3

We define the relation $\mathcal{R}$ on $\mathbb{R}^2$ by :

$$(x,y)\mathcal{R}(x^{'},y^{'}) \Leftrightarrow \; x+y = x^{'} + y^{'}$$

1. Show that $\mathcal{R}$ is an equivalence relation.
2. Find the equivalence class of the couple $(0,0)$.

Exercise 4

We define the relation $\mathcal{T}$ on $\mathbb{R}^2$ by

$$(x,y)\mathcal{T}(x^{'},y^{'}) \Leftrightarrow |x - x^{'}| \leq y^{'} - y.$$

1. Verify that $\mathcal{T}$ is an order relation. Is this order total ?
2. Let $(a,b) \in \mathbb{R}^2$ represent the set $\{(x,y) \in \mathbb{R}^2 / (x,y)\mathcal{T}(a,b)\}$.