Larbi Ben Mh'idi University Oum el Boughi
Academic Year: 2023/2024
Department of Computer Science
Second Year L.M.D-Computer Science
Module: Mathematical Logic
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TD Series No = 02 part 02 (propositional logic : semantics)

Exercise 01:

Let the formula P defined as : $(p \rightarrow (q \rightarrow r)) \rightarrow (r \vee \neg p)$

1. Provide the truth table for the formula P.
2. Indicate whether the formula is valid, satisfiable, or unsatisfiable.
3. Does the formula P have a model? If so, which one?
4. Provide the CNF (Conjunctive Normal Form) and DNF (Disjunctive Normal Form) of the formula P (from the truth table)

Exercise 02:

Establish the truth tables for the following formulas and indicate whether they are valid, satisfiable, or unsatisfiable.

a. $(\neg P \wedge \neg Q) \rightarrow (\neg P \vee R)$

b. $P \wedge (Q \rightarrow P) \rightarrow P$

c . $(P \vee Q) \wedge (\neg P \wedge \neg Q)$

d. $(P \rightarrow Q) \wedge (Q \vee R) \wedge P$

e. $((P \vee Q) \rightarrow R) \leftrightarrow P$

f. $(P \wedge R) \vee (Q \wedge \neg R)$

g. $(\neg P \wedge R) \vee (\neg Q \wedge \neg R)$

Exercise 03:

1. Verify that the formulae below (called paradoxes of material implication) are tautologies:

a) $P \Rightarrow (Q \Rightarrow P)$
b) $\neg P \Rightarrow (P \Rightarrow Q)$.
2. Do you give a translation in natural language?

Exercise 04:

1) Complete the following truth table.

a	b	c	(¬ a ∧ ¬ b) ∨c	(a →b) ∧ (b→c)	(a∨b→c) ⇔(a→b)∧(b→c)

2) Specify whether there is a tautology in the previous table. Justify your answer.
3) Specify if there is an unsatisfiable formula. Justify your answer.
4) Provide a model for (a→b)∧(b→c). Justify your answer.
5) Give all formulas equivalent to a∨b→c.

Exercise 05:

Verify using the truth tables, if

a) $p \Leftrightarrow q \models p \rightarrow q$

b) $p \Leftrightarrow \neg q \models p \rightarrow q$

c) $vrai \models r \rightarrow (s \rightarrow (t \wedge s \rightarrow r))$

d) $\{q \rightarrow (r \wedge s), \neg r \vee \neg s\} \models \neg q$

Exercise 06:

Consider the following reasoning:
When it is sunny, I wear my glasses or I do not go out.
I only stay at home when I do not wear glasses and it is gray.
So if I do not wear my glasses, it is gray.

1. Formalize this reasoning using the following variables:
   S: it is sunny, L: wear my glasses, M: I stay at home.
2. Show by two methods that the reasoning above is correct (valid).

Exercise 07:

Let f be a logical function with 4 logical variables, such that f=1 if and only if the number of variables of f that are '1' is not less than 2.

1. Establish the truth table for f.
2. Provide the conjunctive normal form (CNF) of f and the disjunctive normal form (DNF) of f.