Propositional Logic
order 0 logic

1. **introduction**

In propositional logic, we study the relationships between statements, which are referred to as propositions or formulae or assertions. These relationships can be expressed through logical connectors that allow us to construct syntactically correct formulae through composition.
In other words, propositional logic, also known as Boolean logic, is the simplest form of logic; it consists only of variables and logical connectors. Like all logic systems, it is characterized by:

- **Syntax**: The way formulae are constructed;
- **Semantics**: The meaning of the formulae;
- **Proof System**: How to demonstrate the formulae

2. **Syntax (Formalization)**

2.1 A countable set V = {p, q, r, …..} \ of letters called propositional variables. These are atomic propositions.
2.2 **Atoms**: We will refer to atoms as propositional variables or elementary propositions.
2.3 **Logic operators or connectives**
2.4 **The constants:** {true and false}. It is noted that ⊥\bot⊥ denotes a propositional constant that is always false, and ⊤\top⊤ denotes a propositional constant that is always true.
2.5 **Literal**: A literal is an atom (positive literal) or the negation of an atom (negative literal).
2.6 **Clause**: A clause is a disjunction of literals (p1∨p2∨…∨pn), where the literals can be either positive or negative.

3. **Propositional Language**

The propositional language consists of formulae that represent propositions. Like other languages, the language of propositional calculus is characterized by its syntax and semantics.

Practice Exercise 1

Let's consider the following context: "He likes strawberries and whipped cream." Indicate whether the following propositions are true or false.

1. "He likes strawberries and he likes whipped cream."
2. "He likes strawberries and he does not like whipped cream."
3. "He likes strawberries or he likes whipped cream."
4. "He likes strawberries or he does not like whipped cream."
5. "He likes strawberries, therefore he likes whipped cream."
6. "He likes strawberries, therefore he does not like whipped cream."
7. "He does not like strawberries, therefore he likes whipped cream."
8. "He does not like strawberries, therefore he does not like whipped cream."

Exercise 02 :
Try to determine if the following formulae belong to propositional logic:
Let A,B,C,D be formulae:
a) ((A∨(¬B))∧(C∨D))yes
b) (A∨B)(∧∨C) no
Exercise 03 :
Let p and q be two propositional variables meaning respectively "it is cold" and "it is raining." Write a simple sentence corresponding to each of the following statements:

1. ¬p: "It is not cold."
2. p∧q: "It is cold and it is raining."
3. p∨qp: "It is cold or it is raining."
4. q∨¬p: "It is raining or it is not cold."
5. ¬p∧¬q: "It is not cold and it is not raining."
6. ¬¬q: "It is raining." "It is not true that it is not raining."
7. It is not the case that it is not raining