*Series of exercises 1 (questions marked * left to the students)*

***Exercise 01*:**

Prove the following:

1) $.\forall x; y \in \mathbb{R} : \left||x| - |y|\right| \leq |x + y|$

2) $\forall x; y \in \mathbb{R} : \forall \varepsilon > 0 : xy \leq \frac{x^2}{2\varepsilon} + \frac{\varepsilon}{2} y^2$ (it's called Cauchy's inequality with ε:

3).$(\forall \varepsilon > 0 : |x| < \varepsilon) \Rightarrow (x = 0)$

4).$\forall x; y \in \mathbb{R} : |x| + |y| \leq |x + y| + |x - y|$

5).$(|x + y| = |x| + |y|) \Leftrightarrow (xy \geq 0)$

6)$\forall x_1; x_2 \dots; x_n; y_1; y_2; \dots \dots; y_n \in \mathbb{R}: (\sum_{i=1}^{n} x_i y_i)^2 \leq (\sum_{i=1}^{n} x_i^2)(\sum_{i=1}^{n} y_i^2)$

Exercise 02 :

Prove the following

1) $\forall \varepsilon > 0 ; \exists n \in \mathbb{N}^* : 0 < \frac{1}{n} < \varepsilon.$

2) $\forall x; y \in \mathbb{R}: (x < y) \Rightarrow \big(E(x) \leq E(y)\big)$

3) $\forall x; y \in \mathbb{R}: E(x) + E(y) \leq E(x + y) \leq E(x) + E(y) + 1$

4). $\forall x \in \mathbb{R}: -1 \leq E(x) + E(-x) \leq 0$

5). $Min\ (x; y) = \frac{x+y-|x-y|}{2}\ ; Max(x; y) = \frac{x+y+|x-y|}{2}$

6).$\forall n \in \mathbb{N}^*; \forall x \in \mathbb{R}: E\left(\frac{E(nx)}{n}\right) = E(x)$

specify if possible sup,.inf,.max,.min, for each set of the following :

a) $A = \left\{\frac{2n+1}{n}\ ; n \in \mathbb{N}^*\right\}.$ *c*)* $C = \left\{\frac{1}{n} + \frac{1}{m}\ ; n \in \mathbb{N}^*, m \in \mathbb{N}^*\right\}.$

b) $B = \left\{\frac{1}{x^2+1}; x \in \mathbb{R}\right\}.$ *d*)* $D = \left\{-2 < x + \frac{1}{2x} < 2\ ; x \in \mathbb{R}^*\right\}.$

Exercise 03 :

1) Prove that:

a) If the natural number n is not a perfect square then $\sqrt{n}$ *is irrational*

b If) $r \in \mathbb{Q}$ *and* $x \notin \mathbb{Q}$ *then* $r + x \notin \mathbb{Q}$.

c) If $r \in \mathbb{Q}^*$ *and* $x \notin \mathbb{Q}$ *then* $r + x \notin \mathbb{Q}$.

d) The number $\sqrt{15} + \sqrt{12}$ *is rational (explain)?*

2) Let the set A defined by :$A = \{1 < x < \sqrt{8}\ ; x \in \mathbb{Q}\}$.

.prove that A accepts a lower bound and does not accept an upper bound in Q

3)the equation $x^3 - x + 1 = 0$ doesn't accept solution in Q.

Exercise 04:

1) Let there E and F be two non empty and bounded set prove that :

a). $(E \subseteq F) \Rightarrow (InfF \leq InfE \leq SupE \leq SupF)$

b).$Sup(E \cup F) = Max\{SupE, SupF\}$

c).$Inf(E \cup F) = Min\{InfE, InfF\}$

d) . and $E - F = \{x - y; x \in E$ و $y \in F\}$ $-F = \{-x; x \in F\}$

2) Prove that :

a) .$Sup(E - F) = SupE - InfF$

b) .$Inf(E - F) = InfE - SupF$

c) .$Sup(-F) = -InfF$

d) .$Inf(-F) = -SupF$

3) Let $E \subset \mathbb{R}_+^\star$ we put $\frac{1}{E} = \left\{\frac{1}{x}; x \in E\right\}$ Prove that :

a) .$Inf\frac{1}{E} = \frac{1}{SupE}$

b*) If $InfE \neq 0$ then .$Sup\frac{1}{E} = \frac{1}{InfE}$

Exercise 05* :

:Prove that:

1) $\forall x; y; z \in \mathbb{R}_+^\star (: x + y + z = 1) \Rightarrow \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq 9\right)$.

2) $\forall x; y \in \mathbb{R}: \frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.

3) The number $\frac{\ln 5}{\ln 6}$ Is irrational ?.

4) $\forall x; y \in \mathbb{R}: E(x) + E(y) + E(x + y) \leq E(2x) + E(2y)$

5) $\forall n \in \mathbb{N}: E\left(\sqrt{n} + \sqrt{n+1}\right) = E\left(\sqrt{4n+2}\right)$.