CHAPTER 1
SOIL STRESS

I. DEFINITION ET HYPOTHESES

Continuous media mechanics is a field interested in the ***deformation of solids*** and ***fluid flow***, if we look at matter from «very close» (nanoscopic scale), the matter is granular, made of molecules, but to the naked eye, a solid object seems continuous, That is, its properties seem to vary gradually.

The basis of continuous media mechanics is the study of deformations and phenomena associated with a transformation of a medium. The notion of deformation is used to quantify how lengths were dilated and angles changed in the medium (stresses-deformations).

The study of the behaviour of real materials (steel, concrete, soil, etc.) involves simplifying hypotheses that allow to define two quantities: stresses and deformations.

The continuous media hypothesis consists of considering media whose characteristic properties, that is to say those which are of interest (density, elasticity, etc.) are continuous.

Additional assumptions may be made; thus a continuous medium can be:

- homogeneous: its properties are the same in all respects;
- isotropic: its properties do not depend on the reference point in which they are observed or measured.

The soil is deformed by external stresses from the structure. For practical purposes, and in order to obtain simple formulas for calculating the stresses and strains of the soil, it is assumed that the soil is a continuous medium subject to its weight and overloads of the structure, described by the general equations of the equilibrium of the massifs.

The stresses are induced in the soil by its weight as well as by the overload from the structure. In the latter case, the calculation is based on the theory of elasticity and only covers the most common load cases in projects.

II. CONCEPTS OF CONSTRAINTS

II.1. TOTAL CONSTRAINT

Or a mass of fine soil saturated, homogeneous and isotropic. If the soil is viewed in a global way, it can be assimilated to a continuous medium and study the stresses exerted on a given facet at a given point of this massif.

Is a mass on the surface of which forces are exerted. By cutting this mass in a fictitious plane (P), the surface element "δ S" around the point "M" on the surface "S" is subjected to a force of δ F (fig I.1).

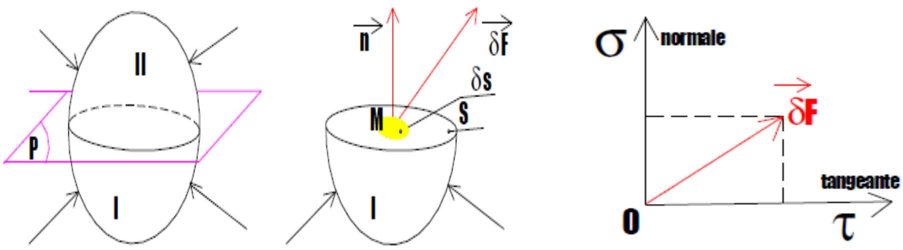


Fig I.1: Constraint in a medium

The constraint at point "M" is the vector $\vec{f} = \frac{\delta \vec{F}}{\delta S}$, This constraint decomposes into a normal constraint σ and a tangential constraint **τ** from which: $\vec{f} = \sigma\vec{n} + \tau\vec{t}$

With: $\vec{n}$ outbound normal unit vector and $\vec{t}$ tangent unit vector.

In soil mechanics, to determine the state of stress around an "M" point in the ground, it is sufficient to know the components of the forces exerted on the faces of a parallelepiped centered around the "M" point and whose edges are parallel to the axes Ox, Oy, Oz.

The state of stresses at point M is defined by a symmetrical matrix called stress tensor:

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

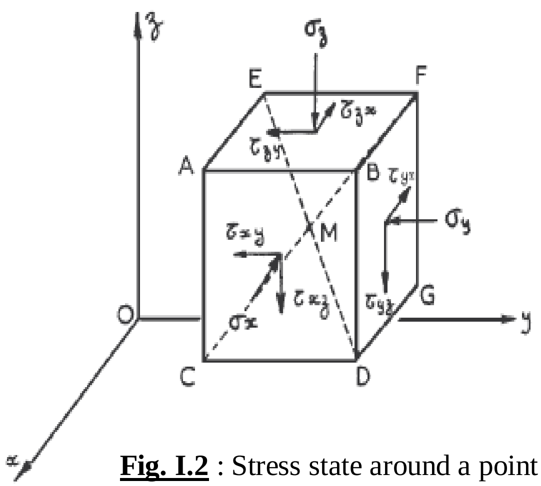


Fig. I.2 : Stress state around a point M

The theory shows that to determine the stresses on all the different facets around a point M, it is sufficient to know at this point the values of the six quantities:

σ_x , σ_y , σ_z , $\tau_{xy} = \tau_{yx}$, $\tau_{zx} = \tau_{xz}$ et $\tau_{zy} = \tau_{yz}$

Among the facets around the M point, there are 3 privileged planes for which the tangential constraint is zero (τ = 0). These 3 planes are called **master plans.**

Their normal directions, main directions **and corresponding constraints,** main constraints, noted:

σ_1: Major main constraint.

σ_2: Intermediate main constraint.

σ_3: Minor main constraint.

With: $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

In other words, by taking these three main directions as a reference, the stress tensor becomes diagonal: $\sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$

II.2. EFFECTIVE CONSTRAINT "TERZAGHI POSTULATE"

Within a soil mass, water pressure or air pressure apply at each point and combine with total stresses to induce local soil behaviour. In saturated (water) soils, it has been accepted since the publication of the Terzaghi «effective stress principle» in 1925 that soil deformation does not depend separately on total stresses and water pressures but on their difference. For this reason, a new type of stress is introduced, called "effective stresses", which are related in the following way to total stresses and pore pressures.

$$\boldsymbol{\sigma = \sigma' + u} \quad \text{et} \quad \boldsymbol{\tau = \tau'}$$

Or:

σ (respectively τ) is the total normal stress (respectively tangential).

σ' (respectively τ') is the normal effective stress (respectively tangential).

u is the interstitial pressure of the fluid ($u = \gamma_w \times h_w$.).

σ' cannot be measured but only calculated.

In dry soils the effective stresses are identical to the total stresses.

III. MOHR CIRCLE

Mr. MOHR had the idea to simply represent, for a given point M of a solid subjected to a given stress f, the distribution of normal or tangential stresses according to the considered facet using a circle called Mohr's circle. Each point described by the circle represents the reading of normal and tangential stress for the facet at an angle θ (θ is the angle between the considered facet and the facet bearing the major stress).

In the two-dimensional case, which is very common in geotechnics, the Mohr circle is the place of the ends of the stress vectors and the main stresses are reduced to two.

III.1 ANALYTICAL METHOD

In the reference system (Ox, O_Z) the stress tensor is written:

$$\sigma = \begin{bmatrix} \sigma x & \tau zx \\ \tau xz & \sigma z \end{bmatrix} \quad \text{with } \tau_{xz} = \tau_{zx}$$

Knowing the stresses on the facets of normal Ox and Oz, one can determine the stresses on any other facet inclined at an angle "θ" - see fig. I.3 –

It should be emphasized that in soil mechanics the following sign convention is adopted:

- $\sigma > 0$ in compression
- $\sigma < 0$ in traction

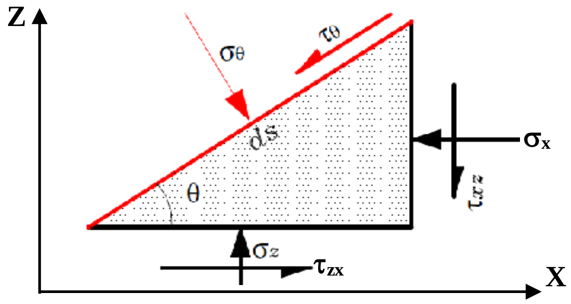


Fig I.3: Two-dimensional stress state.

If the first equilibrium condition is written (the sum of the forces is zero), the state of stress on the inclined plane of "θ" will be as follows:

$$\sigma_\theta = \frac{\sigma x + \sigma z}{2} + \frac{\sigma z - \sigma x}{2} \cos 2\theta - \tau xz \sin 2\theta$$

$$\tau_\theta = \frac{\sigma z - \sigma x}{2} \sin 2\theta + \tau xz \cos 2\theta$$

The locus of stresses in the plane (σ, τ) is defined by the relationship :

$$\left(\sigma\theta - \frac{\sigma x + \sigma z}{2}\right)^2 + \tau^2\theta = \left(\frac{\sigma z - \sigma x}{2}\right)^2 + \tau^2 xz$$

It is the equation of a circle (Mohr's circle) of:

- coordinate center **($(\sigma_x + \sigma_z)/2$, 0).**
- Radius: $R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2}$

The orientation of the main plans is obtained for $\tau_\theta = 0$, or

$$\theta_1 = -\frac{1}{2} arctg \frac{2\tau xz}{\sigma z - \sigma x} \quad \text{et} \quad \theta_2 = \theta_1 + \pi/2$$

There are therefore two main plans, the orientation of which is given by θ_1 and θ_2. The major and minor main constraints are determined from the circle equation:

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2} \quad \text{and} \quad \sigma_3 = \frac{\sigma x + \sigma z}{2} - \sqrt{\left(\frac{\sigma z - \sigma x}{2}\right)^2 + \tau xz^2}$$

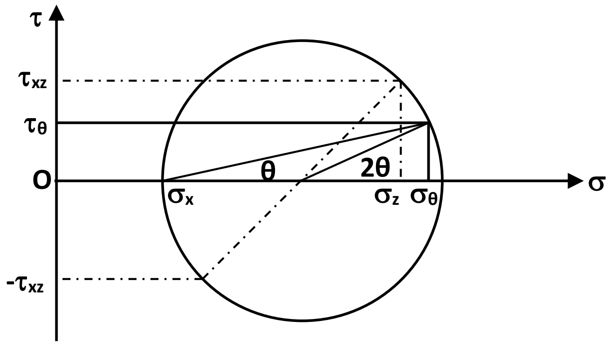


Fig I.4: Mohr circle.

Note that if the x **and** z **directions** are main ($\sigma_x = \sigma_3$; $\sigma_z = \sigma_1$ and $\tau_{xz}=0$) we find:

$$\sigma\theta = \frac{\sigma 1 + \sigma 3}{2} + \frac{\sigma 1 - \sigma 3}{2} \cos 2\theta$$

$$\tau\theta = \frac{\sigma 1 - \sigma 3}{2} \sin 2\theta$$

III.2 GRAPHIC METHOD

The stress state is determined on the plane inclined at an angle θ and whose values of main stresses$\sigma_1$and$\sigma_3$are known (see fig. I.5)

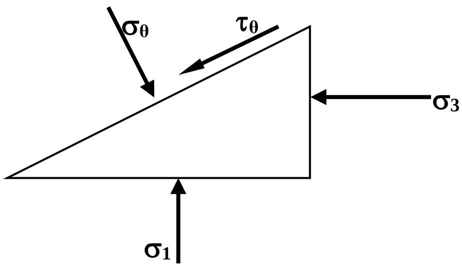


Fig I.5: Stress state on an inclined plane.

The approach used to solve this problem is as follows (see fig. I.6):

- Fromσ_1, we trace a parallel to the plane of σ_1
- Fromσ_3, we trace a parallel to the plane of σ_3
- The intersection of the two planes gives the "P" pole
- From the pole «P», we trace the parallel to the facet on which we want to find the state of constraints (σ_θ and τ_θ)
- The intersection of this line with the circle gives σ_θ and τ_θ.

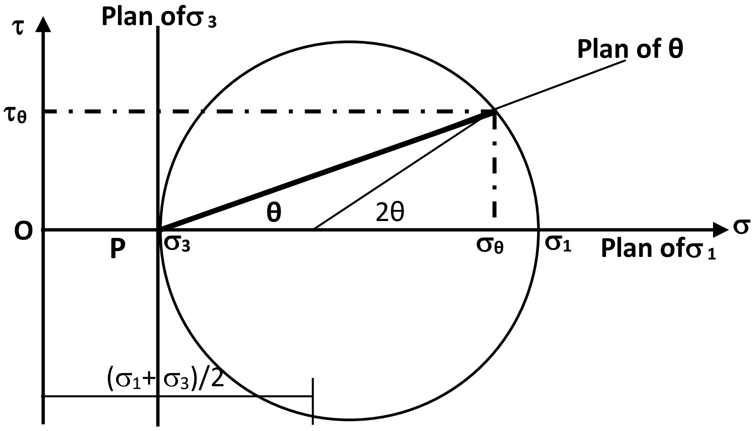


Fig I.6: Mohr's circle (graphical method)

IV. STRESS CALCULATION

IV.1. REAL CONSTRAINT – OVERLAY PRINCIPLE-

The soil is treated as a semi-infinite elastic medium with a horizontal surface. The calculation of stresses in a heavy and loaded mass is based on the use of the principle of superposition (see fig.I.7).

The actual stress (σ_z) at the depth **Z** on a horizontal facet is equal to the sum of the natural stress ($\boldsymbol{\sigma_{v0}}$), due to the weight of the overlying soil and the stress due to overloads ($\Delta\sigma_Z$).

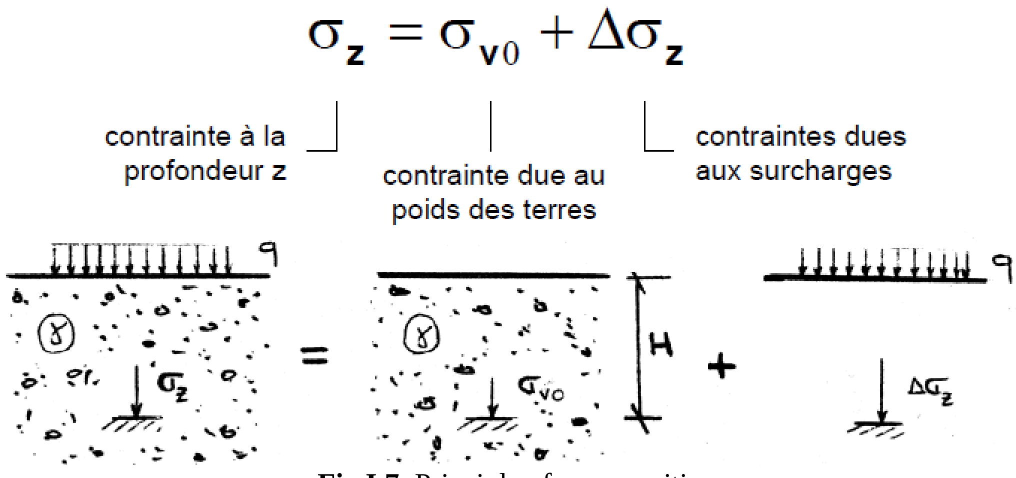


Fig I.7: Principle of superposition.

IV.2. EIGENWEIGHT CONSTRAINTS (GEOSTATIC)

Natural (or geostatic) stress is the stress exerted on a horizontal free surface soil on a horizontal facet before loading; It is generally the weight of the land that overcomes the point considered. The facet considered having its vertical normal the corresponding normal stress is marked$\boldsymbol{\sigma_{v0}}$.

For a soil of density **γ** (in kN/m3),

and at a depth **z** (in m) see fig. I.8, the stress vertical is:

$$\sigma_V = \gamma_z \times z$$

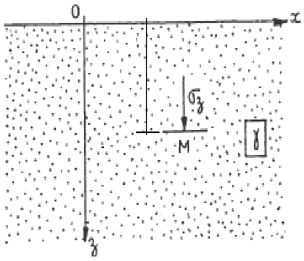


Fig. I.8 : Undefined soil with horizontal surface

In the case of a laminate floor in several layers with different density weights and different heights:

$$\sigma_V = \sum \gamma_{di} \times d_i$$

Example: Let us plot the variation diagrams of **σ** $_v$, **σ'** $_v$ and **u** in relation to the depth

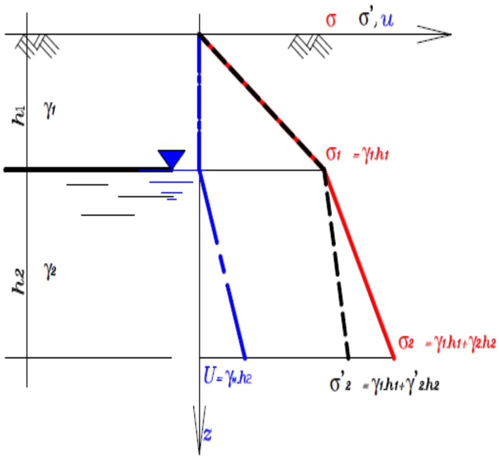


Fig I.9 Diagraphes of Variation of Total, Effective and Interstitial according to the Depth.

IV.3. OVERLOAD STRESS

IV.3.1 Case of a load evenly distributed over the entire surface q

In this case, regardless of the depth **z**, we have:

$$\Delta\sigma_Z = q$$

IV.3.2 Point overload Q

We use the formula of Boussinesq which gives the vertical stress at any point M of a non-weighted elastic medium loaded with a vertical point force Q:

$$\Delta\sigma_z = \frac{3Q}{2\pi}\frac{z^3}{\left(r^2+z^2\right)^{\frac{5}{2}}}$$

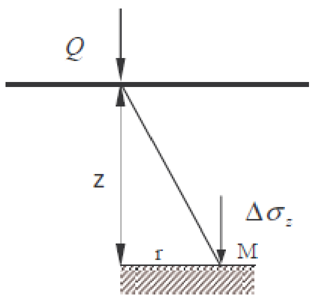


Fig. I.10 : Point load

This relation ship can still be written:

$$\Delta\sigma_z = \frac{Q}{z^2}N \qquad \text{With} \qquad N = \frac{3}{2\pi\left\{1+\left(\frac{r}{z}\right)^2\right\}^{\frac{5}{2}}}$$

The appendix to abaque N° 1 gives the variations of **N** as a function of **r/z.**

IV.3.3 Case of a uniform rectangular load

The stress increase in a semi-infinite medium under the corner of a uniform rectangular distribution (q) is given by the relation:

$\Delta\sigma = k.q$ With q in KN/m^2 and

$k = k(m,n)$ avec $m = \frac{a}{z}; n = \frac{b}{z}$ is a factor of influence with out dimension given in abaque N°2

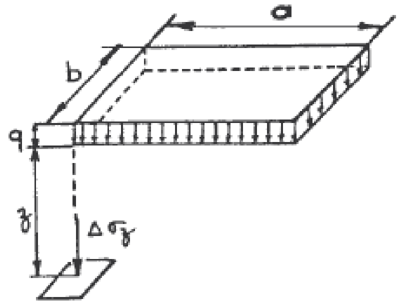
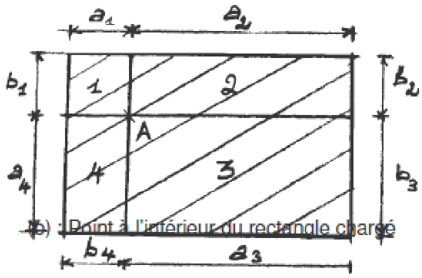


Fig. I.11 : Uniform rectangular load

Example

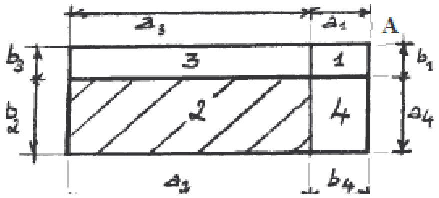
If point A is inside the loaded rectangle

$$\Delta\sigma_z = (k_1 + k_2 + k_3 + k_4)q$$



If point A is outside the loaded rectangle

$$\Delta\sigma_z = (k_{1,2,3,4)} - k_{(3,1)} - k_{(1,4)} + k_1).q$$



IV.3.4 Loading in infinite length fill

The vertical stress **under the corner** of a distribution of loads of infinite length in shape of fill and at depth z (fig. I.12) is given by:

$$\Delta\sigma_z = I.q$$

$I = I\left(\frac{a}{z}, \frac{b}{z}\right)$: coefficient without given dimension in abaque N°3.

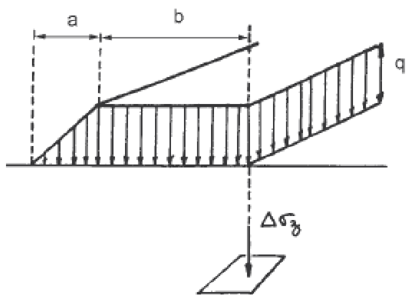
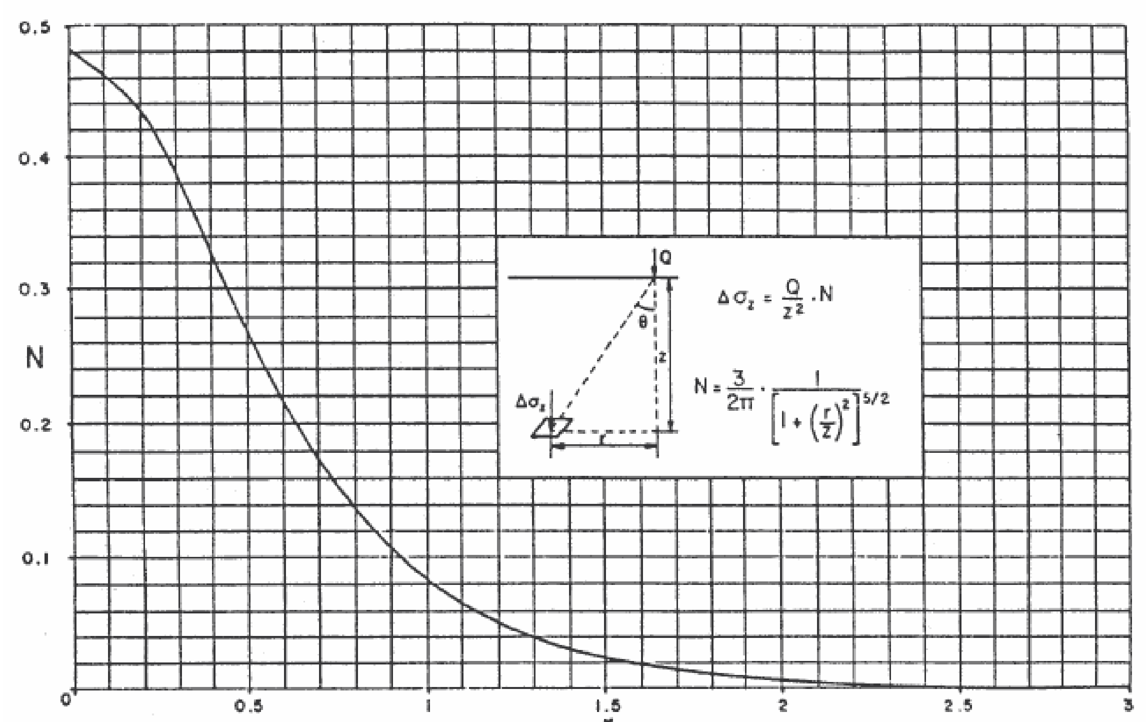


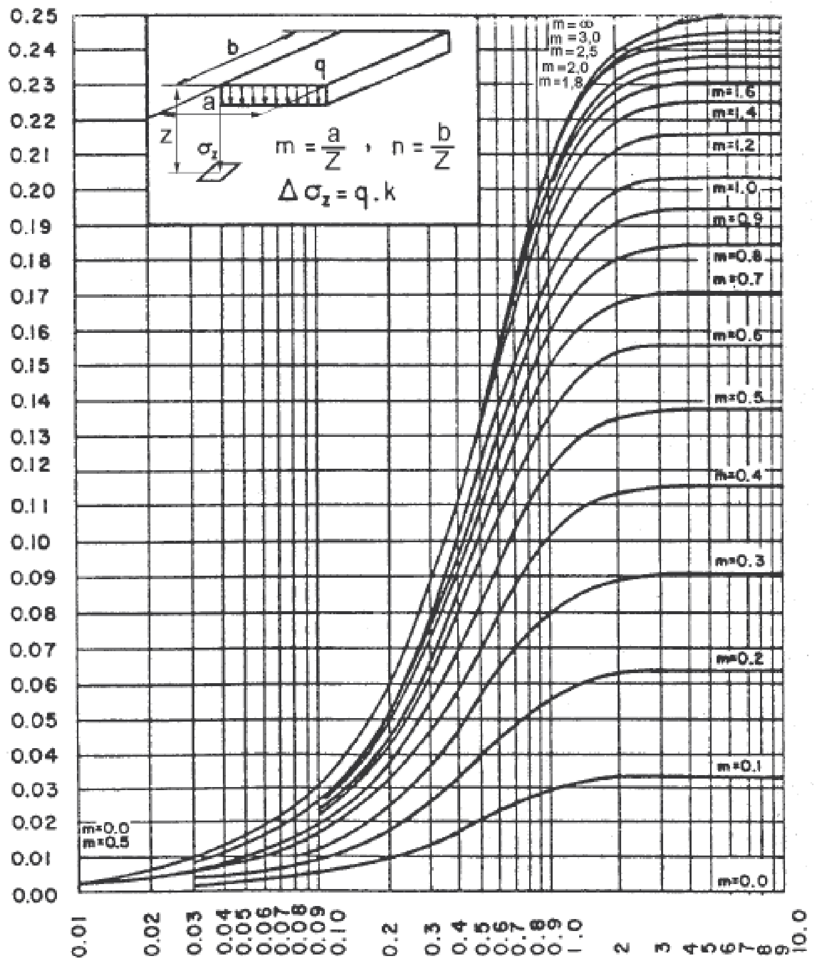
Fig. I.12 : Filling in the fill

Note:

This is a constraint value **under the corner** of a load distribution. Thus, when the backfill to two slopes, do not forget to add the action of the right part to that of the left.



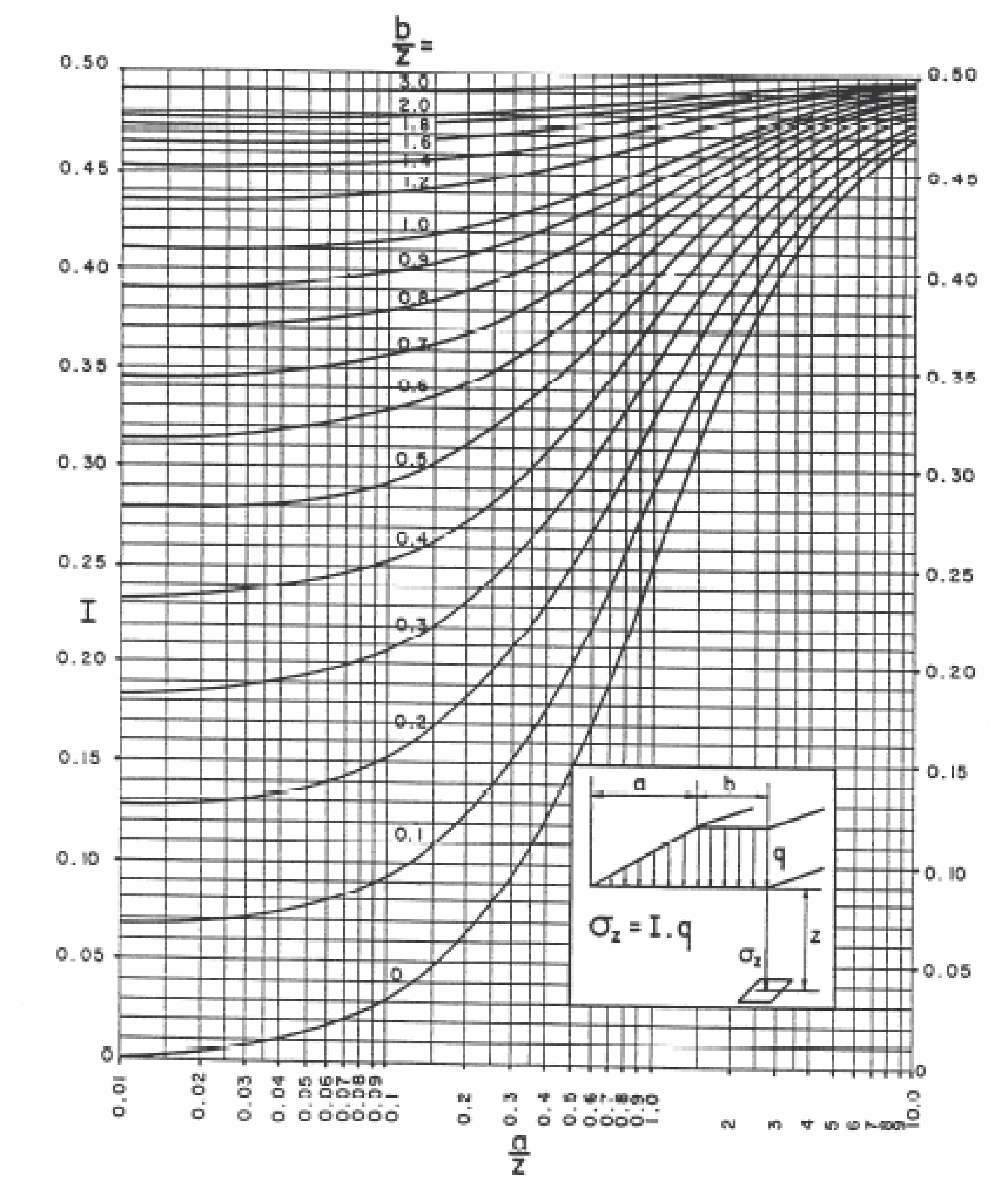
Abaque 1



Note : m et n sont interchangeable

$h = \frac{b}{z}$

Abaque 2



Abaque 3